

Cosmic Strings

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I. DEFINITION OF THE SUBJECT AND ITS IMPORTANCE

Cosmic strings are linear concentrations of energy that form whenever phase transitions in the early universe break axial symmetries [1], as originally shown by Kibble [2, 3]. They are the result of frustrated order in the quantum fields responsible for elementary particles and their interactions. For about two decades, motivation for their study was provided by the possibility that they could be behind the density inhomogeneities that led to the observed large-scale structures in the universe. Precision observations, particularly of the cosmic microwave background radiation, have limited strings to a sub-dominant role in structure formation. Instead, the inhomogeneities appear to be consistent with a period of cosmological inflation, but it turns out that particle-physics models of the early universe that predict a period of inflation very often also predict the generation of cosmic strings at the end of it [4, 5].

More recently, interest has been revived with the realization that there may be strong links between field theory cosmic strings and fundamental strings. The latter are the supposed ultimate building blocks of matter, and in their original context of superstring theory were thought to be microscopic. However, in its modern version—sometimes referred to as M-theory—it is possible and perhaps even mandatory to have macroscopic (cosmological-sized) fundamental strings [6, 7, 8]. Their behavior is expected to be quite similar to that of field theory cosmic strings, although there are some important differences so they may in principle be observationally distinguishable. Being relics of the phase transitions that produced them, cosmic strings provide us with a unique window into the early universe. If they are stable and survive for a significant amount of time (possibly even up to the present day), they may leave an imprint in many astrophysical and cosmological observables, and provide us with information on fundamental physics and the very early universe that would otherwise be inaccessible to us. On the other hand, gaining a quantitative understanding of their properties, interactions, evolution and consequences represents a significant challenge because of their intrinsic complexity. Their non-linearity is particularly noteworthy, with highly non-trivial feedback mechanisms between large (cosmological) and small (mi-

croscopic) scales affecting the network dynamics. Considerable reliance, therefore, must be placed on numerical simulations, which are technically difficult and computationally costly. A complementary approach is the use of analytic or semi-analytic models, usually to describe the large-scale features of the networks.

The basic picture of the cosmological evolution of string networks that has emerged for the simplest (Goto-Nambu) networks is of a scaling solution with about 40 long strings always stretching across each horizon volume plus a population of loops (other string types can lead to a different behaviour). It is then possible to estimate their cosmological implications quantitatively. For example, these strings continuously source gravitational perturbations on sub-horizon scales. The one parameter in these models is the energy scale of the phase transition at which the strings are created. The astrophysical consequences of strings stem from the non-trivial gravitational field around a string [9]. Particles in the vicinity of a static straight string feel no gravitational acceleration, because in general relativity tension is a negative source of gravity and, since tension equals energy per unit length, their effects cancel. The space-time around the string is locally, but not globally, flat. In fact the space is conical, with a deficit angle

$$\alpha = 8\pi \frac{G\mu}{c^4}, \quad (1)$$

where μ is the energy per unit length; the simple way to picture this is to imagine a plane in which an angular wedge α has been removed and the edges glued together. For cosmologically interesting strings the deficit angle ranges from a few seconds of arc to a few millionths of a second of arc.

II. INTRODUCTION

To understand the cosmological evolution and effects of cosmic strings we start in this section with a quick summary of basic cosmology concepts that will be needed later and a description of the simplest type of cosmic strings in which the main features are already apparent. But, first, a warning about units: from now on we will set the speed of light to unity $c = 1$, so we measure distances in light-travel time, and masses in units

of Energy (and viceversa, using $E = mc^2$). Boltzmann's constant is set to unity, so temperature is measured in units of mass/energy (using $E = K_B T$). Finally, we set Planck's constant to unity, $\hbar = 1$, and measure all lengths and masses in units of Planck's length and mass/energy: $l_P = 1.62 \times 10^{-35}$ m, $M_P = 2.18 \times 10^{-8}$ kg = 1.22×10^{19} GeV/c². In these units, Newton's constant is given by $G = M_P^{-2}$.

The early universe is very smooth. To a very good approximation it is a homogeneous and isotropic space-time described by a single variable: the rate of expansion of its three-dimensional spatial sections. In Einstein's general relativity this spacetime is described by the flat Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a^2(t)[d\vec{x} \cdot d\vec{x}] , \quad (2)$$

where \vec{x} are fixed (comoving) spatial coordinates and $a(t)$ is the scale factor that determines the fractional or *Hubble expansion rate*

$$H(t) = \frac{1}{a(t)} \frac{da}{dt} . \quad (3)$$

The time coordinate t is known as cosmological time; to analyse cosmic string evolution we will also need a different time parametrization known as conformal time, τ . They are related by $d\tau = dt/a(t)$, leading to the metric

$$ds^2 = a^2(\tau)[d\tau^2 - d\vec{x} \cdot d\vec{x}] . \quad (4)$$

The age of the universe is currently estimated to be about 13.7 billion years [10]. The universe starts very hot and dense and is cooled by the expansion, with the temperature decreasing as $T(t) \sim a(t)^{-1}$. The Hubble expansion rate is determined by the energy contents of the universe. In a universe dominated by radiation or very relativistic matter (the hottest, earliest stages), the scale factor evolves as $a(t) \sim t^{1/2}$ and the energy density in radiation as $\rho_{\text{radiation}} \sim a(t)^{-4} \sim t^{-2}$. The energy density of non-relativistic matter is inversely proportional to volume $\rho_{\text{matter}} \sim a(t)^{-3}$ and eventually takes over (after about 4000 years), leading to a period of matter domination, during which $a(t) = t^{2/3}$ and therefore $\rho_{\text{matter}} \sim t^{-2}$. More recently –about five billion years ago– we have entered an epoch of accelerated expansion due possibly to a cosmological constant or some unknown form of *dark energy* whose energy density is constant in time $\rho_{\text{dark energy}} \sim \text{const.}$ Dark energy should not be confused with dark matter, an unknown form of matter whose presence we can detect through its gravitational effects but that does not interact with electromagnetic fields and so in particular does not emit light –hence the adjective “dark”–. In the currently accepted cosmological model the energy density in the universe today would be dominated by dark energy (about 74%), followed by about 22% dark matter and only about 4% of regular (baryonic) matter [10]. Dark matter is widely believed to be a particle still to be discovered.

The universe today is far from smooth, but the structure we observe on the scale of clusters of galaxies is consistent with the gravitational collapse of tiny primordial density inhomogeneities $\delta\rho/\rho \sim 10^{-5}$ at the time the cosmic microwave background (CMB) radiation was emitted. The CMB is the oldest radiation we observe, dating back to the time when the universe was only 380000 years old. At this epoch the primordial plasma cooled enough to allow the formation of the first atoms (a process known as recombination), and it became transparent to photons (which is referred to as decoupling). Before that moment, the photons behave like a fluid that is strongly coupled to the protons and electrons. An overdense region in the baryon fluid would like to contract but the photon pressure pushes it back, causing both fluids to oscillate. These oscillations are imprinted in the cosmic microwave background and can be detected today in the form of Doppler peaks in its power spectrum.

The spectrum of density inhomogeneities has been accurately measured in the CMB and found to be near scale-invariant and of the right magnitude to produce the structure we observe. The perturbations to the FRW metric can be classified as *scalar* (overall changes to the Newtonian gravitational potential), *vector* (associated with velocity and/or rotational effects) and *tensor* (transverse traceless perturbations to the spatial metric, such as gravitational waves). Each of these affects the CMB in different ways, so their relative contributions can in principle be observationally distinguished. Finally, Thomson scattering of the anisotropic distribution of the CMB photons is particularly important during decoupling and recombination, and induces a partial linear polarization of the scattered radiation, at a level that is around ten percent of the anisotropy. Detection of this polarization signal is at the borderline of the sensitivity of ongoing experiments at the time of writing, but is expected to become standard with forthcoming experiments.

The energy density of a network of cosmic strings in the linear scaling regime is $\rho_{\text{strings}} \sim t^{-2}$ and therefore it remains a constant fraction of the dominant form of energy during matter or radiation domination. Numerical estimates for the simplest, Goto-Nambu, networks suggest the fraction is around $100G\mu$. Provided the string mass is not close to the Planck scale, this is small enough not to disturb the cosmological evolution; at the same time, for a broad range of values of $G\mu$ this is large enough to be detectable in precision experiments today. Other string types (see section VI) may have larger or smaller fractions or qualitatively different signatures. In particular, networks that do not reach linear scaling may come to dominate the energy density (which rules them out) or to disappear completely.

The simplest field theory model that produces cosmic strings has a single complex scalar field Φ (this is shorthand for a function $\Phi(t, \vec{x})$ with complex values that do not change under coordinate transformations). Let us assume that the Hamiltonian determining the field dy-

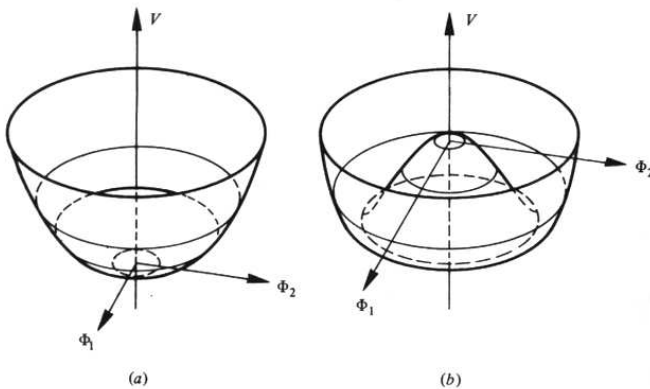


FIG. 1: The effective potential energy V for a simple string-forming field theory model. The (a) and (b) plots correspond to the high and low temperature configurations, respectively. For simplicity the complex field Φ has been split into two real scalar fields, Φ_1 and Φ_2 .

namics is invariant under an axial symmetry such as a phase rotation, $\Phi \rightarrow \Phi e^{i\theta}$. For example, take the potential energy

$$\int d^3x V = \int d^3x \frac{\lambda}{2} (|\Phi|^2 - \eta^2)^2 \quad (5)$$

where λ is a dimensionless coupling constant and η is an energy scale related to the temperature of the symmetry breaking transition. This has a set of degenerate ground states: the minimum of the potential in field space is the circle $|\Phi| = \eta$, known as the *vacuum manifold*. Any configuration $\Phi(t, \vec{x}) = \text{const.} = \eta e^{i\chi}$ with χ real and constant is a possible ground state or *vacuum*, irrespective of the value of the phase χ .

Figure 1 illustrates what happens. At high temperature the field fluctuations are large enough to make the central peak around $|\Phi| = 0$ irrelevant, and the effective potential is symmetric and has a minimum there. As the temperature falls the energy will eventually be too low to permit fluctuations over the peak, at which point the field will tend to settle towards one of the ground states. The random choice of minimum in this condensation process then breaks the original axial symmetry. This is the case, for instance, in superfluid ^4He .

When a large system goes through a phase transition like this, each part of it has to make this random choice, which need not be the same everywhere. The minimization of gradient terms in the energy of the system tends to make it evolve towards increasingly more uniform configurations, but causality (the principle that no information can travel faster than light) imposes that this evolution can only happen at a limited rate. As a result one expects many domains, each with an uncorrelated choice of ground state. Where these domains meet there is some probability of forming linear defects—cosmic strings—around which the phase angle varies by 2π (or possibly multiples thereof). This is the Kibble mechanism. No-

tice that the field vanishes at the string's core, so there is trapped potential energy (as well as gradient energy). These strings are known as *global* strings because the axial symmetry that is broken below the phase transition is “global”, that is, the transformation $\Phi \rightarrow \Phi e^{i\theta}$ is independent of position.

The next step is to consider charged scalar fields interacting with an electromagnetic field. The best known example of a symmetry-breaking transition of this kind is the condensation of Cooper pairs in a superconductor, that has the effect of making photons massive below the critical temperature (in this case the axial symmetry is of the “local” or “gauge” type). The cosmic strings that result are magnetic flux tubes that do not dissipate because the magnetic field is massive outside the string core.

This type of vortex was first discussed by Abrikosov [11] in the context of type II superconductors. Nielsen and Olesen [12] generalized these ideas to the relativistic quantum field theory models used in particle physics, in particular the Abelian Higgs model which is a relativistic version of the Landau-Ginzburg model of superconductivity, governed by the action.

$$S = \int d^4x \left[|\partial_\mu \Phi - iqA_\mu \Phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{2} (|\Phi|^2 - \eta^2)^2 \right]. \quad (6)$$

A_μ is the gauge field and Φ is a complex scalar of charge q ($q = 2e$ in superconductors, where Φ is the Cooper pair wavefunction). The second term is the usual Maxwell action for the electromagnetic field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The energy per unit length of a straight, static string lying on the z -axis is

$$E = \int d^2x \left[|\partial_x \Phi - iqA_x \Phi|^2 + |\partial_y \Phi - iqA_y \Phi|^2 + \frac{1}{2} B^2 + \frac{\lambda}{2} (|\Phi|^2 - \eta^2)^2 \right] \quad (7)$$

where $B = \partial_x A_y - \partial_y A_x$ is the z -component of the magnetic field. Finite energy configurations must have $|\Phi| = \eta$ (the *vacuum manifold* is still a circle) but the phase of Φ is undetermined provided the gradient terms and the magnetic field go to zero fast enough. This condition allows for finite energy solutions $A_t = A_r = A_z = 0$, $\Phi(r, \theta) \sim \eta e^{in\theta}$, $A_\theta(r, \theta) \sim n/(qr)$, as $r \rightarrow \infty$, in which the total magnetic flux in the plane perpendicular to the string is quantized,

$$\int d^2x B = \oint \vec{A} \cdot d\vec{l} = \frac{2\pi n}{q}$$

n is the winding number of the string. If the constants λ and q are such that fluctuations in the scalar field Φ and the gauge field A_μ have equal masses, it is possible to show that the string saturates an inequality of the form

$$\text{Energy per unit length} \geq \text{constant} \times |\text{magnetic flux}|$$

known as the Bogomolnyi bound [13]. In this case, parallel strings at close range exert no force on each other and

there are static multivortex solutions [14]. If the mass of the scalar excitations is lower (higher) than that of the gauge excitations, parallel strings will attract (repel).

More complicated particle physics models—in particular those describing the early universe—involve gauge symmetries that generalize the electromagnetic interaction, mediated by photons, to more complicated interactions such as the electroweak or Grand Unified interactions. The messenger fields that play the role of the photons may be massless in the early universe and become massive following a symmetry-breaking transition, and cosmic strings carry the magnetic flux of these other massive gauge fields (not the electromagnetic field).

From a cosmological point of view, the gauge field has the important effect of making the gradient terms decay exponentially fast away from the string so the energy per unit length of these strings is finite. Abrikosov–Nielsen–Olesen strings have no long-range interactions, so their evolution is dominated by their tension and is well described in the thin string or Goto-Nambu approximation.

Field continuity implies that a string of this kind cannot simply come to an end: it must form a closed loop or extend to infinity, and it cannot break into segments. For this reason, strings, once formed, are hard to eliminate. In the absence of energy loss mechanisms, the strings would eventually dominate the energy density of the universe. On the other hand, the strings can decay into radiation, they may cross and exchange partners, and they may also cross themselves, forming a closed loop which may shrink and eventually disappear. The outcome of these competing mechanisms is that the network is expected to reach a scale-invariant (or *scaling*) regime, where the network’s characteristic length scale is proportional to the size of the horizon. We will discuss string evolution in more detail in Section IV. If a random tangle of strings was formed in the early universe, there would always be some strings longer than the horizon, so a few would remain even today. Because cosmological phase transitions typically happen in the very early universe, cosmic strings contain a lot of trapped energy, and can therefore significantly perturb the matter distribution. To first order there is a single parameter quantifying the effects of strings, its energy per unit length. In the simple relativistic strings, the mass per unit length and the string tension are equal, because of Lorentz invariance under boosts along the direction of the string (but this need not be true for more elaborate models, see section VI. Cosmic strings are exceedingly thin, but very massive. Typically, for strings produced around the epoch of grand unification, the mass per unit length would be of order $\mu \sim 10^{21} \text{kg m}^{-1}$ and their thickness 10^{-24}m . The gravitational effects of strings are effectively governed by the dimensionless parameter $G\mu$, where G is Newton’s constant. For GUT-scale strings, this is 10^{-6} , while for electroweak-scale strings it is 10^{-34} .

III. STRING FORMATION

Spontaneous symmetry breaking is a ubiquitous feature of our theories of fundamental particle interactions. Cosmic strings are formed in many symmetry-breaking phase transitions. If the symmetry is broken from a group G down to a subgroup H , the set of degenerate vacuum or ground states is the manifold $M = G/H$, and the topology of this manifold determines the types of defect that can form. In our previous examples M was a circle; in general, strings can form if M contains closed curves that cannot be contracted within M (the technical term is that M is not simply connected, or that its first homotopy group is non-trivial) [2].

The Kibble mechanism described in section II relates the initial density of strings to the size ξ of the domains over which the field is correlated,

$$\rho_{\text{string}} \sim C \frac{1}{\xi^2}$$

with C a constant of order one reflecting the probability that a strings forms when three or more domains meet. The correlation length cannot grow faster than the speed of light so in the early universe an obvious upper bound on ξ is the size of the horizon at the time of the phase transition. If the dynamics of the phase transition is known, ξ can be estimated more accurately. In a first order phase transition ξ is given by the typical distance between bubble nucleation sites, which depends on the nucleation rate. In second order phase transitions ξ depends on the critical exponents and the rate of cooling through the critical temperature, T_c , as shown by Zurek [15, 16].

Vortex lines or topological strings can therefore appear in a wide range of physical contexts, from cosmic strings in the early universe through disclinations in room-temperature nematic liquid crystals, to magnetic flux tubes in some superconductors and vortex lines in low-temperature superfluid helium. These systems provide us with a range of opportunities to test aspects of the cosmic string formation and evolution scenario experimentally.

The Kibble mechanism in first order transitions was confirmed in experiments on nematic liquid crystals [17, 18]. The Kibble-Zurek scenario for second order transitions has been experimentally verified in Superfluid ^3He [19, 20] and in Josephson Tunneling Junction arrays [21].

The He-3 experiments in a rotating cryostat in Helsinki also confirmed the scale invariance of the initial distribution of loops, $n(R) \sim R^{-4}$, where $n(R)dn$ is the number density of loops with radii between R and $R + dR$, as predicted by Vachaspati and Vilenkin [22].

More recently, the formation of a defect network following the annihilation of $^3\text{He-A}$ / $^3\text{He-B}$ boundary layers has been observed [23]. The precise type of defects is still under investigation but this system constitutes an

interesting analogue to the formation of strings from the annihilation of branes in brane inflation scenarios.

There are also a few systems where the string density disagrees with the Kibble-Zurek predictions. In ^4He [24], the reasons are understood: the strings are fuzzy and the network does not survive long enough to be detected [25]. In the case of superconducting films the results are somewhat inconclusive [26] and also it is not completely clear what the expected density of flux quanta should be after a temperature quench; an alternative formation mechanism with different vortex clustering properties has been proposed in [27]. In fact the formation of defects in systems with gauge fields is clearly very relevant to cosmology but is still not completely understood (see [28] for a recent discussion).

IV. STRING EVOLUTION

The motion of a cosmic string with worldsheet coordinates σ^a and background space-time coordinates x^μ in a metric $g_{\mu\nu}$ is obtainable from a variational principle applied to the Goto-Nambu action [29, 30]

$$S = \mu \times \text{Area} = \mu \int d\tau d\sigma |det g_{ab}|^2 = \mu \int d\tau d\sigma \left| det \begin{pmatrix} \dot{x}^\rho \dot{x}^\nu g_{\rho\nu} & \dot{x}^\rho x'^\nu g_{\rho\nu} \\ x'^{\rho'} x'^{\nu'} g_{\rho'\nu'} & x'^{\rho'} x^\nu g_{\rho'\nu} \end{pmatrix} \right|^{1/2} \quad (8)$$

where μ is again the string mass per unit length, and with dots and primes respectively denoting derivatives with respect to the time-like (τ) and space-like (σ) coordinates on the world-sheet. g_{ab} is called the induced metric. We are interested in strings in a FRW background space-time (see equation (4)) and can choose worldsheet coordinates that make the induced metric diagonal

$$\sigma^0 = \tau, \quad \dot{\mathbf{x}} \cdot \mathbf{x}' = 0, \quad (9)$$

The choice of conformal time coordinate simplifies the microscopic evolution equations, although as we shall see later on physical time is a more natural choice for the macroscopic evolution (see Section II for definitions of the two time choices). It is also useful to define the coordinate energy per unit σ ,

$$\epsilon^2 = \frac{\dot{\mathbf{x}}'^2}{1 - \dot{\mathbf{x}}^2}. \quad (10)$$

Then the usual variational techniques can be used to show that the microscopic string equations of motion are

$$\ddot{\mathbf{x}} + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}(1 - \dot{\mathbf{x}}^2) = \frac{1}{\epsilon} \left(\frac{\mathbf{x}'}{\epsilon} \right)' \quad (11)$$

and

$$\dot{\epsilon} + 2\epsilon\frac{\dot{a}}{a}\dot{\mathbf{x}}^2 = 0. \quad (12)$$

For simplicity we are neglecting effects such as cusps and a frictional force due to particle scattering (which

for heavy strings is only relevant during a transient period very early in the network's evolution). The first one is just a wave equation with a particular damping term (provided by the expansion of the universe). The damping also has the effect of reducing the coordinate energy per unit σ .

As was mentioned earlier the expansion of the universe stretches the strings, so in the absence of energy loss mechanisms their energy would grow with the scale factor and the string network would eventually become the dominant component of the universe's energy density—which would be in conflict with observational results. Such decay mechanisms do exist (at least for the simplest models), being ultimately due to radiation losses and to the fact that whenever strings interact they will reconnect [31, 32]. In particular closed loops may be formed, and these subsequently oscillate and eventually decay. This decay is thought to be mainly into gravitational radiation, but other forms of radiation are also produced very efficiently.

Provided the decay rate is high enough, the network will not have pathological consequences, but will instead reach a linear scaling solution, where the string density is a constant fraction of the background density and on large scales the network looks the same (in a statistical sense) at all times. Scaling is in fact an attractor solution, as has been shown both using numerical simulations and analytic models. Physically, the reason for this is that if one has a high density of strings then the number of string interactions increases and therefore loop production becomes more efficient and the decay rate increases. Conversely if the density is too low then there are few interactions and the decay rate is correspondingly lower. Numerical simulations confirm this broad picture, but also reveal that string evolution is a complex non-linear process, involving non-trivial interactions between various different scales.

There have been thus far two generations of numerical simulations of Goto-Nambu cosmic string networks in expanding universes. The first (Albrecht and Turok [33], Bennett and Bouchet [34], Allen and Shellard [35]) dates from around 1990, at the peak of the interest in cosmic strings as possible seeds for the large-scale structures we observe today. In the last few years, the renewed interest in strings in the context of models with extra dimensions led to a second generation of simulations (Martins and Shellard [36], Ringeval et al. [37], Vanchurin et al. [38]), which build upon previous knowledge and exploit the dramatic improvements in hardware and software in the intervening decade and a half to achieve a much higher resolution.

A different approach is provided by full field theory simulations [39]. These are closer to the microphysics of the defects and provide unique information on the interactions of the defects and their energy loss mechanisms, but their shorter dynamic range means that they are not optimal for understanding the non-linear feedback mechanisms between widely different scales which affect the

dynamics of the network. From this point of view they play a very important role as calibrators, both for Goto-Nambu simulations and for analytic models. One can also carry out Minkowski space simulations (either of Goto-Nambu or field theory type). Neglecting the expansion of the universe is numerically desirable, since such simulations are much easier to implement and evolve much faster. However, the expansion plays a non-trivial role in the network dynamics, so these results should not be naively extrapolated to realistic cosmological scenarios.

Initial conditions for the numerical simulations are usually set up using the Vachaspati-Vilenkin algorithm [22]. One often adds to this random initial velocities, since these tend to enhance the rate of relaxation. All simulations agree on the broad, large-scale features of string networks, and in particular on the fact that the linear scaling solution is an attractor for the evolution. In Goto-Nambu simulations, the initial fraction of the total energy in the form of closed loops is around 20%, but in the linear scaling regime this fraction is around 50% or even slightly more. On the other hand, in field theory simulations this fraction tends to be somewhat smaller.

The first-generation simulations suggested a dynamical picture where the long-string network lost energy to large, long-lived loops, with sizes of order the correlation length. Refinements had each loop self-intersecting into around 10 daughter loops, but loop production from the long strings was essentially monochromatic. The second-generation simulations, however, reveal a quite different picture. Large loops do self-intersect (and indeed the number of daughter loops produced by each one seems to be around 20), but there is also a direct production of large quantities of small loops from slow-moving long-string with fractal-like substructure. In other words, the loop production is in fact bi-modal. All three second-generation simulations agree on this broad picture, though not on which of the two loop production scales is dominant.

The second-generation simulations present some tentative evidence for the scaling of small-scale features of the network. An open question is whether or not this is expected to happen, given that gravitational backreaction (which would provide a characteristic scale) is not included in any of the network simulations carried out to date. One possible explanation stems from the fact that large loops are not scaled up versions of small loops. Indeed, small loops tend to be nearly circular, whereas large loops are not only far from circular but even far from planar. In other words, the self-intersection probability for a given loop depends on its size, and this may be sufficient to dynamically select a preferred scale. Incidentally, the loop fragmentation processes in these networks highlight the fact that there is a steady flow of energy from large to small scales which is entirely analogous to a Richardson cascade in turbulence. (In this case energy enters via long strings falling inside the horizon, and leaves via radiative decays.)

Figure 2 shows some relevant quantities characterizing

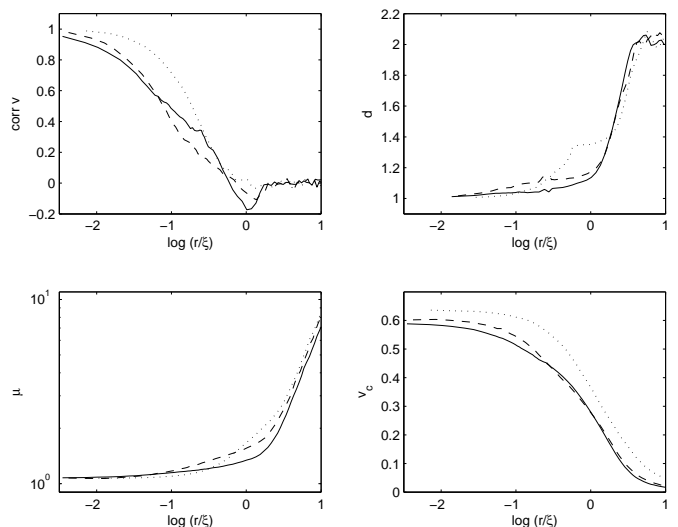


FIG. 2: Characteristic small-scale properties of cosmic string networks in the linear scaling regime, for matter (solid lines), radiation (dashed) and Minkowski spacetime (dotted) runs. In all plots the horizontal axis represents the logarithm of the physical lengthscale relative to the correlation length of the network. The simulations leading to these results are described in [36]. Top panels show the correlation function for the velocity vectors and the fractal dimension, bottom panels show coarse-grained mass per unit length and coherent velocity. The plotted quantities are described in the main text.

Goto-Nambu string networks in the linear scaling regime. These are always plotted as a function of scale, relative to the network's correlation length. The top left panel shows the correlation function for the velocity vectors. A first striking feature is that in the expanding universe cosmic string velocities are anti-correlated on scales around the correlation length (which is smaller than but comparable to the causal horizon), but such a feature is not present in Minkowski space. This anti-correlation is the result of a 'memory' of the network for recent reconnection events, and it is ultimately due to the damping effect of expansion. The top right panel depicts the fractal dimension of the network: this interpolates between $d = 1$ (straight segments) on small scales and $d = 2$ (Brownian network) on large scales, but it does so in a non-trivial way (which is again different depending on whether or not there is expansion) and over a wide range of scales. The fractal dimension evolves with time, decaying on any given physical scale: the strings continually become smoother on any scale, so as to minimize energy. Finally, the bottom panels show the renormalized (or 'coarse-grained') string mass per unit length on the left, and the corresponding coherent velocity on the right panel—notice that the effect of expansion is to reduce the velocities on any given scale.

The technical difficulty and computational cost of numerical simulation provide strong motivation for alternative analytic approaches, which essentially abandon

the detailed statistical physics of the string network to concentrate on its thermodynamics. The best example is the velocity-dependent one-scale (VOS) model [40], which builds on previous work by Kibble and Bennett and has demonstrated quantitative success when compared with both field theory and Goto-Nambu numerical simulations. The ‘one-scale’ assumption is that the network has a single characteristic lengthscale, which coincides with the string correlation length and the string curvature radius. This is an approximation which can be tested numerically.

The first assumption in this analysis is to localize the string so that we can treat it as a one-dimensional line-like object. This is clearly a good assumption for gauged strings, such as magnetic flux lines, but may seem more questionable for strings possessing long-range interactions, such as global strings or superfluid vortex lines. However, good agreement between the VOS model and simulations has been found in both cases. The second step is to average the microscopic string equations of motion to derive the key evolution equations for suitable macroscopic quantities, specifically its energy E and RMS velocity v defined by

$$E = \mu a(\tau) \int \epsilon d\sigma, \quad v^2 = \langle \dot{\mathbf{x}}^2 \rangle = \frac{\int \dot{\mathbf{x}}^2 \epsilon d\sigma}{\int \epsilon d\sigma}. \quad (13)$$

Notice that the energy is an ‘extensive’ quantity but the RMS velocity is an averaged quantity (and the averaging is weighted by the coordinate energy ϵ). In keeping with the above coordinate choices, the microscopic quantities (on the right-hand side of both equations) are defined in terms of conformal time, but it turns out that the macroscopic evolution that we are now considering is best described in terms of physical time—please refer to the cosmology review for the explicit relation between the two.

Any string network divides fairly neatly into two distinct populations, *viz.* long (or ‘infinite’) strings and small closed loops. In the following we will focus on the long strings. The long string network is a Brownian random walk on large scales and can be characterised by a correlation length L , which can be used to replace the energy $E = \rho V$ in long strings in our averaged description, that is,

$$\rho \equiv \frac{\mu}{L^2}. \quad (14)$$

A phenomenological term must then be included to account for the loss of energy from long strings by the production of loops, which are much smaller than L . A *loop chopping efficiency* parameter \tilde{c} is introduced to characterise this loop production as

$$\left(\frac{d\rho}{dt} \right)_{\text{to loops}} = \tilde{c} v \frac{\rho}{L}. \quad (15)$$

In this approximation, we would expect the loop parameter \tilde{c} to be a constant; comparison with numerical simulations suggests $\tilde{c} \sim 0.23$.

From the microscopic string equations of motion, one can then average to derive the evolution equation for the correlation length L ,

$$2 \frac{dL}{dt} = 2HL(1 + v^2) + \tilde{c}v, \quad (16)$$

where H is the Hubble parameter defined in eq. (3). The first term in (16) is due to the stretching of the network by the Hubble expansion which is modulated by the red-shifting of the string velocity, while the second is the loop production term. One can also derive an evolution equation for the long string velocity with only a little more than Newton’s second law

$$\frac{dv}{dt} = (1 - v^2) \left(\frac{k(v)}{L} - 2Hv \right). \quad (17)$$

The first term is the acceleration due to the curvature of the strings and the second is the damping term from the Hubble expansion. Note that strictly speaking it is the curvature radius R which should appear in the denominator of the first term. In the present context we are identifying $R = L$. The function $k(v)$ is the *momentum parameter*, defined by

$$k(v) \equiv \frac{\langle (1 - \dot{\mathbf{x}}^2)(\dot{\mathbf{x}} \cdot \mathbf{u}) \rangle}{v(1 - v^2)}, \quad (18)$$

with $\dot{\mathbf{x}}$ the microscopic string velocity and \mathbf{u} a unit vector parallel to the curvature radius vector. For most relativistic regimes relevant to cosmic strings it is sufficient to define it as follows:

$$k_r(v) = \frac{2\sqrt{2}}{\pi} \frac{1 - 8v^6}{1 + 8v^6}, \quad (19)$$

while in the opposite case ($v \rightarrow 0$), we have the non-relativistic limit $k_0 = 2\sqrt{2}/\pi$.

Scale-invariant attractor solutions of the form $L \propto t$ (or $L \propto H^{-1}$) together with $v = \text{const.}$, only appear to exist when the scale factor is a power law of the form

$$a(t) \propto t^\beta, \quad 0 < \beta = \text{const.} < 1. \quad (20)$$

This condition implies that

$$L \propto t \propto H^{-1}, \quad (21)$$

with the proportionality factors dependent on the expansion rate β . It is useful to introduce the following parameters to describe the relative correlation length and densities, defining them respectively as

$$L = \gamma t, \quad \zeta \equiv \gamma^{-2} = \rho t^2 / \mu. \quad (22)$$

By looking for stable fixed points in the VOS equations, we can express the actual scaling solutions in the following implicit form:

$$\gamma^2 = \frac{k(k + \tilde{c})}{4\beta(1 - \beta)}, \quad v^2 = \frac{k(1 - \beta)}{\beta(k + \tilde{c})}, \quad (23)$$

where k is the constant value of $k(v)$ given by solving the second (implicit) equation for the velocity. It is easy to verify numerically that this solution is well-behaved and stable for all realistic parameter values.

If the scale factor is not a power law, then simple scale-invariant solutions like (23) do not exist. Physically this happens because the network dynamics are unable to adapt rapidly enough to the changes in the background cosmology. An example of this is the transition between the radiation and matter-dominated eras. Indeed, since this relaxation to a changing expansion rate is rather slow, realistic cosmic string networks are strictly speaking *never* in scaling during the matter-dominated era. Another example is the onset of dark energy domination around the present day. In this case, the network is gradually slowed down by the accelerated expansion, and asymptotically it becomes frozen in comoving coordinates. The corresponding scaling laws for the correlation length and velocity are $L \propto a$ and $v \propto a^{-1}$.

Despite its success in describing the large-scale features of string networks, the VOS model has the shortcoming of not being able to account for the small-scale features developing on the strings as the network evolves, as clearly shown by numerical simulations. This small-scale structure is in the form of wiggles and kinks, and can be phenomenologically characterized by its fractal properties, as we have sketched above. As a first analytic simplification, the string wiggles can be characterized through a renormalized string mass per unit length that is larger than the bare (Goto-Nambu) mass. This effectively corresponds to considering a model with a non-trivial equation of state (the relation between the string tension and the mass per unit length), which turns out to be one among a larger class of models known as elastic string models. This kind of description has interesting parallels with the coarse-graining approaches that are typical of condensed matter.

A more radical approach is to explicitly abandon the one-scale assumption. This is done in the three-scale model [41], which distinguishes between the characteristic lengthscale (which is simply a measure of the total string energy in a given volume) and the persistence length (which is defined in terms of the invariant length along the string and corresponds to the correlation length or inter-string distance). Additionally there is a third lengthscale which approximately describes a typical scale of the small-scale wiggles. This kind of description is in principle highly flexible, though this can be considered a blessing and a curse. The downside is that one is forced to introduce a large number of (almost free) phenomenological parameters over which one has limited control even when comparing the model with simulations.

Having said that, the three-scale model does confirm, at least qualitatively, the expectations for the behavior of string networks. Scaling of the large scales (in this case the characteristic and persistence lengths) is found to be an attractor, just as in the VOS model. Depending on the behavior of small-scale structures, the two large

length scales may reach scaling simultaneously or the former may do so before the latter—a behavior that has been seen in numerical simulations. As for the behavior of the small-scale structures, their evolution timescale is typically slower, and generically they only reach scaling due to the effects of gravitational backreaction (not included in numerical simulations). In the absence of gravitational backreaction, scaling of the small-scale characteristic length is contingent of the removal of a sufficiently large amount of small-scale structure from the long strings by radiation and loop production, which in the model is controlled by a parameter whose detailed behavior is not known.

Finally, an interesting and rather different approach starts out with the assumption that there is a range of scales where stretching due to the expansion is the dominant dynamical effect, even on scales well below the cosmological horizon. A sufficient condition for this is that one is assuming that the rate of string intercommutations is fixed in horizon units. This turns out to be sufficient to allow the construction of a statistical-type description based on two-point correlation functions [42]. Their results are to a first approximation dependent on a critical exponent which physically is related to the coherent string velocity on a given scale. Comparison with numerical simulations shows, as expected, that the best agreement is found around and just below the horizon scale.

A second assumption is that loop production at those scales is sufficiently localized to be describable as a perturbation. When loop production is thus folded into the analysis, the picture that ultimately emerges is of a complicated fragmentation cascade. In particular, this model provides supporting evidence for the two-population loop distribution picture outlined above and clearly seen in high-resolution simulations. There is a population of correlation-length sized loops, produced by direct long-string intercommutation, and a second population with sizes a few orders of magnitude below (quite possibly near the gravitational backreaction scale) and due to loop fragmentation. Whether or not the smoothing provided by gravitational radiation is necessary to yield scaling of the loop sizes is again not entirely clear at the moment, but it is in principle a question for which this formalism could provide an answer.

V. ASTROPHYSICAL AND COSMOLOGICAL CONSEQUENCES

As was mentioned in Section I, the spacetime around a straight cosmic string is flat. A string lying along the z -direction has an equation of state $p_z = -\rho$, $p_x = p_y = 0$ and therefore there is no source term in the relativistic version of the Poisson equation for the Newtonian gravitational potential

$$\nabla^2 \phi = 4\pi G(\rho + p_x + p_y + p_z) = 0. \quad (24)$$

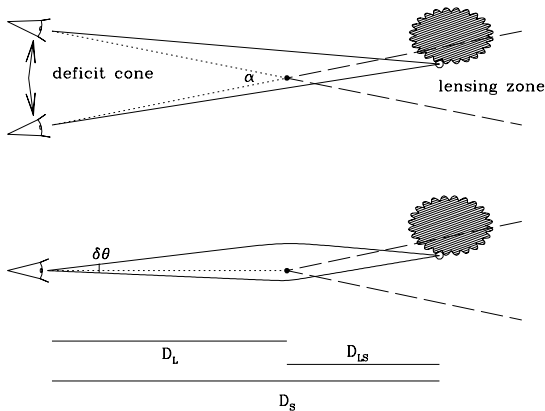


FIG. 3: An illustration of the mechanism behind lensing by cosmic strings. The thick black dot represents a cosmic string perpendicular to the page. The spacetime metric around the string can be obtained by removing the angular wedge of width α and identifying the edges. An observer can thus see double images of objects located on a certain zone behind the string. This zone is enclosed by the dashed lines, while the solid lines depict light rays and the angular separation of the two images, $\delta\theta$, will depend on the distances of the source and the observer to the string as well as on the deficit angle. Reprinted, with permission, from [43].

A straight string exhibits no analogue of the Newtonian pull of gravity on any surrounding matter. However, this does not mean the string has no gravitational impact at all. On the contrary, we will see that a moving string has dramatic effects on nearby matter or propagating microwave background photons. It is not difficult to derive the spacetime metric about such a straight static string [9]. It has the simple form

$$ds^2 = dt^2 - dz^2 - dr^2 - r^2 d\theta^2, \quad (25)$$

which looks like Minkowski space in cylindrical coordinates, except for the fact that the azimuthal coordinate θ has a restricted range $0 \leq \theta \leq 2\pi(1 - 4G\mu)$. That is, the spacetime is actually conical with a global deficit angle

$$\alpha = 8\pi G\mu; \quad (26)$$

where an angular wedge of width α is removed and the remaining edges identified.

This deficit angle implies that the string acts as a cylindrical gravitational lens, creating double images of sources behind the string (such as distant galaxies), with a typical angular separation $\delta\theta$ of order α and no distortion [44]. This is illustrated in Figure 3. A long string would yield a distinctive lensing pattern. We should expect to see an approximately linear array of lensed pairs, each separated in the transverse direction. In each lensing event the two images would be identical and have essentially the same magnitude. (Except if we happen

to see only part of one of the images.) This is a very unusual signature, because most ordinary gravitational lenses produce an odd number of images of substantially different magnitudes. A number of string lensing event candidates have been discussed in the past, but no confirmed one is currently known.

However, the above simple picture is complicated in practice by the fact that cosmic strings are not generally either straight or static. Whenever strings exchange partners kinks are created that straighten out only very slowly, so we expect a lot of small-scale structure on the strings. Viewed from a large scale, the effective tension and energy per unit length will no longer be equal. Since the total length of a wiggly string between two points is greater, it will have a larger effective energy per unit length, U , while the effective tension T , the average longitudinal component of the tension force, is reduced, so $T < \mu < U$. This means that there is a non-zero gravitational acceleration towards the string, proportional to $U - T$. Moreover, the strings acquire large velocities, generally a significant fraction of the speed of light, which introduces further corrections to the deficit angle.

Another effect is the formation of over-dense wakes behind a moving cosmic string [45]. When a string passes between two objects, these are accelerated towards each other to a velocity

$$u_{\perp} = 4\pi G\mu v, \quad (27)$$

where v is the string velocity. Matter therefore collides in a sheet-like structure, leaving a wake behind the moving string. This was the basic mechanism underlying the formation of large-scale structures in cosmic string models. This model has significant attractions, such as the early formation of nonlinear structures, and one can get a good match to the observed galaxy power spectrum in models with a large cosmological constant. However, as we shall discuss, it fails to reproduce the power spectrum of CMB anisotropies observed by COBE, WMAP and other experiments; cosmic strings, therefore, can only play a subdominant role in structure formation (albeit still significant, at the ten to twenty percent level). Cosmic strings create line-like discontinuities in the cosmic microwave background signal [46, 47]. For the same reason that wakes form behind a cosmic string, the CMB source on the surface of last scattering is boosted towards the observer, so there is a relative CMB temperature shift across a moving string (a red-shift of the radiation ahead of it, and a blue-shift of that behind), given by

$$\frac{\delta T}{T} \sim 8\pi G\mu v_{\perp}. \quad (28)$$

where v_{\perp} is the component of the string velocity normal to the plane containing the string and the line of sight. This is known as the Kaiser-Stebbins effect. This simple picture is again complicated in an expanding universe with a wiggly string network and relativistic matter and radiation components. The energy-momentum tensor of

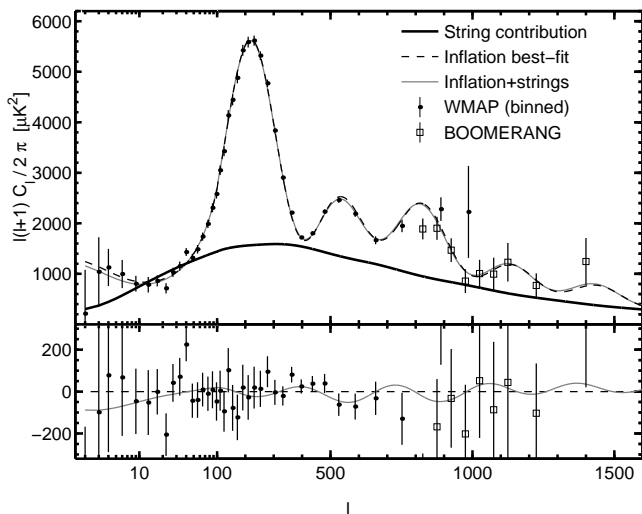


FIG. 4: The CMB temperature power spectrum contribution from cosmic strings, normalized to match the WMAP data at $\ell = 10$, as well as the best-fit cases from inflation only (model PL) and inflation plus strings (PL+S). These are compared to the WMAP and BOOMERANG data. The lower plot is a repeat but with the best-fit inflation case subtracted, highlighting the deviations between the predictions and the data. Reprinted, with permission, from [49].

the string acts as a source for the metric fluctuations, which in turn create the temperature anisotropies. The problem can be recast using Green's (or transfer) functions which project forward the contributions of strings at early times to today [48]. The actual quantitative solution of this problem entails a sophisticated formalism to solve the Boltzmann equation and then to follow photon propagation along the observer's line of sight. At the time of writing, the most recent comparisons [49] between full-sky maps of cosmic string-induced anisotropies and WMAP data yield a cosmological constraint on the models with

$$G\mu < \text{few} \times 10^{-7}, \quad (29)$$

with only a weak dependence on the background cosmology—in particular, on the magnitude of the cosmological constant.

Apart from their scale-invariance (which follows from the network's attractor scaling solution discussed in the previous section), cosmic defect-induced fluctuations appear to be the antithesis of the standard inflation paradigm, because they are causal or active (they are generated inside the horizon and over an extended period), there are also large vector and tensor contributions, and they are distinctly non-Gaussian. All these characteristics leave clear signatures in the cosmic microwave background angular power spectrum, chief of which is a much broader primary Doppler peak and little evidence of secondary oscillations. Unlike inflation, active defect sources act incoherently with extra large-scale power from the vectors and tensors. Moreover, their isocurva-

ture nature provides a partial explanation for why the broad primary peak ends up at larger multipoles (typically $\ell \sim 300$ as opposed to $\ell \sim 200$ in a flat cosmology. Isocurvature perturbations correspond to equal and opposite perturbations in the radiation and matter densities—as opposed to equal fractional perturbations in the number densities of the two components for adiabatic perturbations—. On the basis of knowledge from present simulations, therefore, cosmic defects alone are extremely unlikely to have been the seeds for large-scale structure formation. However, they cannot be ruled out entirely. For example, admixtures of inflationary power spectra with significant cosmic defect contributions (at a level around 20%, see Figure 4) do provide a satisfactory fit to present data. This is interesting among other reasons because it is the sort of level at which the non-Gaussian signatures of cosmic strings should still be discernible, although their distinct line-like discontinuities are only clearly identifiable on small angular scales around a few arc minutes.

Accelerated cosmic strings are sources of gravitational radiation [50]. Consequently, a network of long strings and closed loops produces a stochastic gravitational wave background [51] over a wide range of frequencies and with a spectrum which (at least to a first approximation) has equal power on all logarithmic frequency bins. Another distinctive signal would come from the cusps, the points at which the string instantaneously doubles back on itself, approaching the speed of light. Such an event generates an intense pulse of gravitational and other types of radiation, strongly beamed in the direction of motion of the cusp [52]. If massive cosmic strings do indeed exist, both these pulses and the stochastic background are likely to be among the most prominent signals seen by the gravitational-wave detectors now in operation or planned, in particular LIGO and LISA.

A stringent, though indirect, limit on the string energy per unit length comes from observations of the timing of millisecond pulsars. Gravitational waves between us and a pulsar would distort the intervening space-time, and so cause random fluctuations in the pulsar timing. The fact that pulsar timing is extremely regular places an upper limit on the energy density in gravitational waves, and hence on the string scale. The upper limit [53] is of order $G\mu < 10^{-7}$, though there is still considerable uncertainty because this depends on assumptions about the evolution of small-scale structure.

Although gravitational waves are thought to be the main decay byproduct of the evolution of the simplest cosmic string networks, direct decay into particle radiation is extremely efficient and there are claims that it could be the dominant energy-loss mechanism responsible for scaling [39]. In more complicated models, there are certainly other decay channels. If the strings are global (rather than local), then they will preferentially produce Goldstone bosons instead. In axion models, these Goldstone bosons acquire a small mass and become the axions (a prime dark matter candidate). One can es-

timate the number density of axions using analytic models of cosmic string evolution such as the VOS model. In another class of string models known as superconducting (since they have additional current-like degrees of freedom), then the decay products can include electromagnetic radiation.

Finally, we should mention the claims that cosmic strings could be responsible for a number of high energy astrophysical and cosmological enigmas, including ultra-high energy cosmic rays, gamma ray bursts, and baryogenesis (the creation of the matter-antimatter asymmetry of the universe). Since cosmic defects can produce high energy particles, they could contribute to the observed cosmic ray spectrum, notably at ultra-high energies $E \geq 10^{11} \text{GeV}$ where the usual acceleration mechanisms seem inadequate [54]. Many ideas have been explored, such as particle emission from cosmic string cusps, but most have been found to produce a particle flux well below current observational limits. However, among the interesting scenarios deserving further study are those with hybrid defects (such as monopoles connected by strings) or vortons.

VI. FIELD THEORY STRINGS WITH MORE DEGREES OF FREEDOM

Kibble's original idea was to consider strings in grand unification scenarios, in which the strong and electroweak forces become unified at an energy scale of around 10^{15-16}GeV . More recent studies have shown that practically any viable Supersymmetric Grand Unified Theory has a pattern of symmetry breaking transitions that leads to the possibility of cosmic string formation at some point in its history [5]. In particular, in these models the inflationary phase generically ends with a phase transition at which strings are produced. More recent studies suggest that this is also a feature of brane inflation models.

So far we have discussed the field theory realization of the simplest model of cosmic string, the Abrikosov–Nielsen–Olesen string, in which the mass per unit length equals the tension and there is no internal structure apart from the magnetic field. In fact, the situation can be much more complex in the early Universe, and realistic particle physics models lead to networks with a much richer phenomenology. The added complexity makes these networks much harder to study, whether by analytic or numerical methods, and consequently they are not as well understood as the simplest case.

We can only give here a brief description of the possible complications. The list below is not complete and furthermore there are strings that fit more than one category. For a more detailed discussion we refer the reader to the reviews by Vilenkin & Shellard [1], Hindmarsh & Kibble [55], Carter [56], and Achúcarro and Vachaspati [57], where references to the original literature can also be found.

A. Wiggly strings; varying tension string networks; cycloops

The name wiggly strings is sometimes used to refer to any type of string whose mass per unit length is different from its tension. We already mentioned in Section V that small structure (wiggles) on the string produces a renormalized effective mass per unit length $U > \mu$ and an effective tension $T < \mu$. There are other effects that can affect the mass and tension, for instance the presence of currents along the strings.

A particular kind of small structure is found in extra dimensional models. If spacetime has more than the three spatial dimensions we observe, strings may be able to wrap around the extra dimensions in different ways leading to a renormalized four-dimensional tension and mass per unit length. The effective tension can of course vary along the strings. In extreme cases, the extra dimensional wrapping effects concentrate around certain points along the string which behave like 'beads' (see hybrid networks below) and are called cycloops.

As discussed towards the end of Section IV, the additional degrees of freedom (which can be thought of as a mass current) make the evolution of the networks highly non-trivial. The one-scale assumption is no longer justified: the correlation length, inter-string distance and string curvature radius become distinct lengthscales. Depending on the exact interplay between the bare strings and the mass current wiggles, these lengthscales can evolve differently, and some of them might be scaling while the others are not. The presence of extra dimensions provides a further energy flux mechanism (as energy may be lost into or gained from the extra dimensions) which will affect the string dynamics, but at the time of writing its exact effects have not been studied in detail.

B. Non-topological / embedded / electroweak / semilocal strings

In the Abrikosov–Nielsen–Olesen case, the scalar field is zero at the core of the string, and the symmetry is unbroken there. The zero field is protected by the topological properties of the vacuum manifold (the non-contractible circle) and the string is called topological. In more realistic models, the criterion for topological string production is a non-simply connected vacuum manifold, however complicated. These strings are unbreakable and stable.

On the other hand there are examples in which there is no topological protection but the strings are nevertheless stable. The scalar field configuration at the core can be deformed continuously into a ground state, so these non-topological strings can break, their magnetic flux can spread out, or be converted to a different type of flux. But whether this happens is a dynamical question that depends on the detailed masses and couplings

of the particles present, on the temperature, etc.

The best studied examples of non-topological strings look like Abrikosov–Nielsen–Olesen strings ‘embedded’ in a larger model such as the Glashow–Salam–Weinberg model of electroweak interactions [58, 59]. These *electroweak* strings carry magnetic flux of the Z boson, but the strings would only be stable for unphysical values of the Z boson mass. They are closely related to *semilocal* strings, another example of embedded strings where the symmetry breaking involves both local and global symmetries intertwined in a particular way. For low scalar mass these can be remarkably stable.

In general, non-topological strings are not resilient enough for the networks to survive cosmological evolution. If the strings are unstable to spreading their magnetic flux, the network will not form. If the strings are breakable the network may form initially but it will quickly disappear (see hybrid networks below for a concrete example). A remarkable exception to this rule are semilocal strings with very low scalar mass: the network forms as a collection of segments which then grow and reconnect to form longer strings or loops. These evolve like a network of Abrikosov–Nielsen–Olesen strings plus a small population of segments and there is some evidence of scaling [60].

C. Dressed / superconducting strings / vortons

In realistic particle physics models, a stable string will trap in its core any particles or excitations whose mass is lower inside due to the interactions with the scalar field. These *dressed* strings have a more complicated core structure. In extreme cases, the mass of these trapped particles is zero in the core and they lead to persistent currents along the strings, which are then known as *superconducting* [61]. In some cases, the decay of a loop of superconducting string can be stopped by these currents, leading to long-lived remnants called *vortons* [62] that destroy scaling; typical vortons will either dominate the energy density of the universe (contrary to observations) or contribute to the dark matter if they are sufficiently light.

D. Hybrid networks

Hybrid networks contain more than one type of defect, such as for instance strings of different kinds or composite defects combining strings, monopoles and/or domain walls.

1. Composite defects:

The production of strings may be accompanied by the production of other defects such as monopoles or domain walls, before or afterwards, that change the behaviour of

the network as a whole. These networks can have radically different scaling properties—in particular, linear scaling may not exist at all. Consider for instance a sequence of breakings of the form $G \rightarrow H \rightarrow K$ in which a symmetry group G first breaks down to a subgroup H which subsequently breaks to an even smaller subgroup K at a lower temperature. Two cases are particularly relevant for strings:

The first breaking produces stable magnetic monopoles, the second confines—totally or partly—the magnetic field to flux tubes (strings) leading to a network of monopoles connected by strings. This can happen either as string segments, with monopoles at the ends, which eventually contract and disappear or as a network of strings carrying heavy “beads” (the monopoles) which can lead to a scaling solution.

The first breaking produces stable strings, the second makes domain walls attached to the strings (e.g. in *axion* models). The network is made of pancake-like structures that contract under the wall tension and eventually disappear, although in some cases there may be long-lived remnants.

2. Non-abelian / (p, q) strings

Another type of hybrid network contains different types of strings whose intercommutation leads to three-point junctions and bridges. These networks are also very different from the simplest ones but the current consensus is that they also seem to reach a scaling solution during cosmological evolution.

In the non-abelian case, the magnetic flux carried by the string is not just a number but can have different internal “orientations”. These become relevant when the strings cross, limiting the ways in which they can reconnect.

Hybrid networks containing several interacting string types are also found in superstring models (see next section). The most interesting type, usually referred to as (p, q) strings, contains two types of string each carrying different type of flux that is separately conserved: fundamental and solitonic or D-strings, roughly corresponding to electric and magnetic flux tubes. The numbers p and q refer to the units of each kind of flux carried by the strings. Since the mass per unit length depends on these fluxes, (p, q) networks are expected to have a hierarchy of different tensions, as well as junctions and bridges. In fact, junctions and bridges will also form in any model in which parallel strings have an attractive interaction, such as Abrikosov–Nielsen–Olesen strings with extremely low scalar to vector mass ratios.

The existence of string junctions and the hierarchy of string tensions make the evolution of these networks considerably more complicated than that of the simple Goto–Nambu strings. Relatively simple analyses suggest that the heavier strings with gradually decay into the lighter ones, and scaling is eventually reached for the strings at

the low end of the spectrum (the heavier ones eventually disappear), although this is still under discussion. Naive expectations that the network might be slowed down to non-relativistic speeds and eventually freeze have so far not been supported by the (admittedly simplistic) numerical simulations performed so far. Further work is needed to understand the general conditions under which scaling is (or is not) an attractor.

VII. COSMIC SUPERSTRINGS

Superstring theory is to date the only candidate model for a consistent quantum theory of gravity that includes all other known interactions. In string theory, the fundamental constituents of nature are not point-like particles but one-dimensional “strings” whose vibrational modes produce all elementary particles and their interactions. Two important features of the theory are supersymmetry (a symmetry between bosonic and fermionic excitations that keeps quantum effects under control) and the presence of extra dimensions above the four spacetime dimensions that we observe.

It is not yet known how to formulate the theory in its full generality but some weak-coupling regimes are well understood. In these, the fundamental strings live in a 10-dimensional spacetime, of which 6 dimensions are “compactified”, resulting in an effective 4-dimensional spacetime we live in. There is another regime, M-theory, in which the fundamental objects are two-dimensional “membranes” and the background spacetime is 11-dimensional. These regimes are related to one another by duality transformations that interchange the role of fluctuation quanta and non-perturbative, soliton-like states (branes), so the expectation is that all regimes are different limits of a unique, underlying theory usually referred to as superstring/M-theory, or just M-theory for short.

Before the discovery of D-branes, the “solitons” of superstring theory, the question of whether fundamental superstrings could ever reach cosmological sizes was analysed and the possibility discarded [63]. There were two main problems. First, the natural mass per unit length of fundamental strings is close to the Planck scale and would correspond to deficit angles of order 2π , which would have been observed. Second, the strings were inherently unstable to either breaking or –depending on the type of string– becoming the boundary of domain walls that would quickly contract and disappear. The discovery of branes and their role in more exotic compactifications where the six compact dimensions have strong gravitational potentials (and redshifts) have changed this picture. It is now believed that networks of cosmic superstrings could be a natural outcome of brane-antibrane annihilation, especially if the branes are responsible for a period of cosmic inflation [7, 64, 65, 66].

An important difference with previous scenarios is that these strings are located in regions of the compactified

dimensions with very strong gravitational redshift effects (“warping”) that reduce the effective mass per unit length of the strings to a level with deficit angles in the region of 10^{-12} to 10^{-7} , compatible with current observations. Another important difference is a much lower probability that the strings intercommute when they cross, estimated to be 10^{-3} to 10^{-1} , depending on the type of strings. The lower intercommutation rates lead to much denser networks. Estimates of the corresponding enhancement in the emission of gravitational radiation by cusps puts these strings in a potentially observable window by future gravitational wave detectors [67, 68].

The networks are hybrid, consisting of fundamental strings and D-strings, the latter being either one-dimensional D-branes or perhaps the result of a higher dimensional D-brane where all but one dimension are wrapped around some “holes” (cycles) in the compactified space. There may also be cycloids.

As in the case of hybrid field theory strings, whether or not superstring networks eventually reach a scaling regime is an open question. Analytic studies and numerical simulations of simplified cases suggest that scaling is certainly possible, though contingent on model parameters that at the time of writing are not well understood. In this case, in addition to the presence of junctions and a non-trivial spectrum of string tensions, a third factor can affect to the evolution of these networks. If the strings are actually higher-dimensional branes partially wrapped around some extra dimensions, then energy and momentum can in principle leak into or out of these extra dimensions [69]. Since the effective damping force affecting the ordinary and extra dimensions is different, one might generically expect that this will be the case. Depending on its sign and magnitude, such an energy flow can in principle prevent scaling, either by freezing the network (if too much energy leaks out) or by making the strings dominate the universe’s energy density (if too much energy leaks in, though this is less likely than the opposite case). In this sense, a somewhat delicate balance may be needed to ensure scaling. At a phenomenological level, further work will be required in order to understand the precise conditions under which each of these scenarios occurs. At a more fundamental level, it is quite likely that which of the scenarios is realized will depend on the underlying compactifications and/or brane inflation models, and that may eventually be used as a discriminating test between string theory realizations.

VIII. FUTURE DIRECTIONS

One of the most exciting prospects is the discovery of magnetic-type CMB polarization (usually referred to as B-modes) as this would reveal the presence of vector and/or tensor modes. Cosmic string models may be further constrained in the near future because B-modes are predicted to have amplitudes comparable to the electric-type E-modes (at large angular scales). At high reso-

lution, one could also hope to observe defects directly through the B-mode signal, against a relatively unperturbed background. Conversely, the detection of vector modes would provide strong evidence against inflation without cosmic defects. Polarization data will also strongly constrain a significant isocurvature contribution to the mainly adiabatic density fluctuations. Isocurvature perturbations can be a signature of more complicated physics during inflation, such as the effects of two or more scalar fields, or the formation of defects at the end of inflation.

Ongoing and future CMB experiments, especially at high resolution, will be probing the degree of Gaussianity of the primordial fluctuations. The detection of significant and unambiguous non-Gaussianity in the primary CMB signal would be inconsistent with simple (so called single field slow-roll) inflation. More general inflationary models can accommodate certain types of non-Gaussianity, and one can also envisage non-Gaussianity from excited initial states for inflation. It is interesting to note that given the existing bounds on $G\mu$, current CMB experiments do not have the sensitivity or resolution to detect cosmic string signatures directly, in particular the Kaiser-Stebbins effect in CMB maps. However, with high-resolution sensitivities becoming available in the near future, direct constraints (or detections) will be possible. This is just one example of the interesting new science that future high-resolution CMB experiments might uncover in the years ahead. In particular, ESA's Planck Surveyor [See www.rssd.esa.int/Planck/], scheduled for launch in late 2008, may be able to provide significant breakthroughs.

A deeper understanding of the evolution and consequences of string networks will require progress on both numerical simulations and analytic modellings. At the time of writing there is still no numerical code that includes all the relevant physics, even for the simplest (Goto-Nambu) strings. Inclusion of gravitational back-reaction is particularly subtle, and may require completely new approaches. The expected improvements in the available hardware and software will allow for simulations with much longer evolution timespan and spatial resolution, which are needed in order to understand the non-linear interactions between large and small scales all the way down to the level of the constituent quantum fields. This in turn will be a valuable input for more detailed analytic modelling, that must accurately describe

the non-trivial small-scale properties of the string networks as well as the detailed features of the loop populations. Better modelling is also needed to describe more general networks—three crucial mechanisms for which at present there is only a fairly simplistic description are the presence of junctions, a non-trivial spectrum of string tensions, and the flow of energy-momentum into extra dimensions.

At a more fundamental level, a better understanding of the energy loss mechanisms and their roles in the evolution of the networks is still missing [70] and it will require new developments in the theory of quantum fields out of equilibrium. Such theoretical developments are also needed to understand defect formation in systems with gauge fields, and could be tested experimentally in superconductors.

The early universe is a unique laboratory, where the fundamental building blocks of nature can be probed under the most extreme conditions, that would otherwise be beyond the reach of any human-made laboratory. Cosmic strings are particularly interesting for this endeavour: they are effectively living fossils of earlier cosmological phases, where physical conditions may have been completely different. The serendipitous discovery of cosmic defects or other exotic phenomena in forthcoming cosmological surveys would have profound implications for our understanding of cosmological evolution and of the physical processes that drove it. The search continues while, in the meantime, the absence of cosmic string signatures will remain a powerful theoretical tool to discriminate between fundamental theories. The possibility that something as fundamental as superstring theory may one day be validated in the sky, using tools as mundane as spectroscopy or photometry, is an opportunity that neither astrophysicists nor particle physicists can afford to miss.

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